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<th>Question</th>
<th>Points Earned</th>
<th>Points Possible</th>
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<td>Bonus</td>
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1. Determine the number of edges, $m$, contained in each of the following graphs.
   (a) $P_n$  
   (b) $C_n$
   (c) $K_n$  
   (d) $K_{m,n}$
   (e) $W_n$  
   (f) a tree with $n$ vertices
   (g) the $k$-cube $Q_k$  
   (h) $N_n$
2. For each part, give an example of a graph $G$ with the desired properties. If no such graph exists, explain why not.

(a) $G$ is Hamiltonian but not Eulerian.

(b) $G$ is Eulerian but not Hamiltonian.

(c) $G$ is connected, contains a cutvertex, and contains a cutset of cardinality 3.
(d) Every edge of $G$ is a bridge, but $G$ is not a tree.

(e) $G$ has degree sequence $(1, 2, 3, 4, 5)$.

(f) $G$ is a simple graph with degree sequence $(1, 3, 3, 3)$. 


3. Determine a minimum weight spanning tree for the following weighted graph.

4. Determine a shortest path from $A$ to $G$ using the shortest path algorithm.
5. Theorem 9.1 gave five characterizations of a tree. List three of these.
Let $T$ be a graph with $n$ vertices. Then the following statements are equivalent:

(i) $T$ is a tree.
(ii) 
(iii) 
(iv) 

6. Let $G$ be a connected graph. What can you say about

(a) an edge of $G$ that appears in every spanning tree?

(b) an edge of $G$ that appears in no spanning tree?
7. (a) For which \( n \) is the graph \( P_n \) bipartite? Explain your answer.

(b) For which \( n \) is the graph \( C_n \) bipartite? Explain your answer.

(c) For which \( n \) is the graph \( W_n \) bipartite? Explain your answer.

Bonus: How many spanning trees does \( K_{2,s} \) have?